Corrections to the Elements of the Orbit of Juno.
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My attention having been drawn to the large and increasing errors of the computed places of *Juno*, as given in the *Nautical Almanac* (amounting to some 3′ in geocentric longitude in 1887), I have endeavoured in the present paper to deduce corrections to the assumed elements of the orbit, so as to obtain elements which shall represent the observed places of the planet with sufficient accuracy for, I hope, some years to come.

The observations of *Juno* which are discussed in this paper are those published in the volumes of *Greenwich Observations* from 1864 to 1887 inclusive. The discussion therefore embraces observations made at the Paris Observatory during the years 1864 to 1878, as well as those made at Greenwich during the whole period. Le Verrier's Tables of the Sun were first used in the *Nautical Almanac* for the year 1864, and it has been assumed that the tabular places of the Sun are sensibly accurate throughout the years with which we are here concerned.

The first step in the discussion of the observations, both Greenwich and Paris, is to reduce them to a uniform system. That adopted for the R.A.'s is the standard system of R.A.'s deduced from 12-hour groups of clock-stars for 1872·0, printed on pp. 21 and 22 of the Introduction to the *Greenwich Nine-Year Catalogue*, and which has been used in the reduction of the Greenwich observations for 1878 to 1888 inclusive. For the years 1864 to 1869 the Greenwich R.A.'s depend on those of the First Seven-Year Catalogue, whilst for the years 1872 to 1877 they depend on those of the Second Seven-Year Catalogue. The following table (deduced from the comparisons printed on page 8 of the above-mentioned Introduction) gives the corrections applicable to the R.A.'s observed at Greenwich during the specified periods to reduce them to the adopted system:—

R.A.	1864—1869.	1870—1877.
h h	s	s
0–3	+0·02	+0·02
3–6	− ·01	+ ·01
6–9	− ·02	·00
9–12	− ·01	+ ·01
12–15	·00	+ ·01
15–18	·00	+ ·01
18–21	+ ·02	+ ·03
21–24	+ ·03	+ ·03

Observations made since 1878·0 require no correction.

In the reduction of the Paris observations of R.A. a "provisional" standard catalogue (constructed in 1854) was used

during the period 1864–1871, with different corrections each year, deduced from then recent observations; whilst during the period 1872–1878 the corrections to the provisional catalogue were deduced from observations extending over the years 1856–1869. It was found, however, that the corrections given in the following table would reduce the Paris observations made throughout the whole period 1864–1878 to the adopted system with sensible accuracy:—

R.A.	Corr. to Paris.	R.A.	Corr. to Paris.
h h	s	h h	s
0–3	+0·02	12–15	+0·01
3–6	− ·01	15–18	+ ·02
6–9	− ·01	18–21	+ ·03
9–12	·00	21–24	+ ·03

With regard to the N.P.D.'s observed at Greenwich during the years 1864–1882, it is necessary to correct them for the law for R.–D.—viz. $a+b'$ sin z cos ^2z, assumed during those years, so as to make them depend on the law $a+b$ sin z. This has been done by assuming that the mean Z.D. at which reflection observations were made during those years (and from which, therefore, the values of a and b' were deduced) was 25°, and that therefore $b=\cdot 8\times b'$. It is also necessary to correct observations made during 1864–1876 by the quantities given in the table on p. 27 of the Addendum to the Introduction to the *Nine-Year Catalogue*, to reduce them to the refractions of Bessel's Tabulæ, and to colatitude 38° 31′ 21″·90. It is further necessary to correct observations made during 1868–1874, for errors due to wear in the micrometer-screws of the transit-circle, by the quantities given on p. 14 of the Introduction to the *Nine-Year Catalogue*. The following table gives the corrections to the observed N.P.D.'s corresponding to the N.P.D. of *Juno*, at the time of opposition, in the different years (exclusive of the correction for wear of screws, which was applied to each individual observation):—

Year.	N.P.D.	Corr.	Year.	N.P.D.	Corr.
	°	″		°	″
1864	94	−0·3	1874	94	+1·2
1865	94	−0·7	1876	87	+1·0
1867	86	+0·3	1877	94	+0·7
1868	92	+1·3	1878	94	+0·6
1869	95	+1·2	1880	86	+0·3
1871	87	+1·0	1881	92	+0·1
1872	90	+1·1	1882	95	+0·2
1873	95	+1·3			

Observations made since 1883·0 require no correction.

The Paris observations in N.P.D. depend on the assumed places of standard stars taken from the corrected "provisional" catalogue referred to above. From a discussion of the places of stars in this catalogue, situated between N.P.D. 70° and 110°, it was found that the following corrections were applicable to Paris observations, between these limits of N.P.D., made during the period 1864–1878, to reduce them to the adopted system of N.P.D.'s:—

R.A.	Corr. to Paris.	R.A.	Corr. to Paris.
h h	$''$	h h	$''$
0–3	+0.4	12–15	+0.4
3–6	+0.3	15–18	+0.6
6–9	+0.2	18–21	+0.7
9–12	+0.2	21–24	+0.5

The publication of Paris observations of minor planets in the volumes of *Greenwich Observations* was intermitted after 1879; and there are no Paris observations of *Juno* printed in the Greenwich volumes after 1878.

The requisite corrections, deduced from those given in the foregoing tables, have been applied to the quantities given in the section "Errors of the Tabular Heliocentric Places of the Planets," printed in successive volumes of *Greenwich Observations*. The corrected equations connecting the error of tabular heliocentric longitude (δl), the error of tabular heliocentric ecliptic north polar distance (δb), and the error of projected radius vector of *Juno* ($\delta\rho$) are given below, together with the mean dates of the groups from which the normal places have been formed, and the number of observations in each group.

No.	Mean Date.	Equations.		No. of Obs.
1	1864 June 10	$-8''.44 = +1.462\delta l'' + 3102''\delta\rho$,	$\delta b'' = +0''.04 - 6546''\delta\rho$	5
2	25	$-8.75 = +1.418 \quad +11728$	$= -0.40 - 6294$	5
3	July 6	$-6.84 = +1.360 \quad +16835$	$= -0.16 - 5862$	2
4	1865 Sept. 2	$-27.31 = +1.769 \quad -9819$	$= -3.98 - 1756$	4
5	13	$-27.10 = +1.788 \quad +6490$	$= -4.75 - 892$	8
6	27	$-25.83 = +1.729 \quad +27421$	$= -4.35 + 335$	8
7	1867 Mar. 1	$-22.47 = +1.574 \quad -3316$	$= +4.71 + 1354$	5
8	27	$-22.57 = +1.477 \quad +18293$	$= +3.70 + 375$	5
9	1868 May 7	$-16.89 = +1.436 \quad -3383$	$= +1.93 - 5377$	1
10	17	$-15.70 = +1.441 \quad +2288$	$= +1.97 - 5480$	4
11	25	$-16.56 = +1.426 \quad +6730$	$= +1.21 - 5442$	4
12	June 6	$-16.32 = +1.379 \quad +12640$	$= +1.36 - 5186$	4
13	1869 July 17	$-31.16 = +1.573 \quad -8243$	$= -3.66 - 7444$	4
14	29	$-31.73 = +1.594 \quad +4905$	$= -3.18 - 7439$	4
15	Aug. 5	$-30.45 = +1.583 \quad +9421$	$= -3.53 - 7283$	3

No.	Mean Date.	*Equations.*					No. Ob
16	1869 Aug. 13	$-29''{\cdot}49=$	$+1{\cdot}549\delta l''$	$+16597''\delta\rho,$	$\delta b''=-4''{\cdot}49$	$-6974''\delta\rho$	
17	1871 Jan. 26	$-59{\cdot}94=$	$+1{\cdot}787$	$+9813$	$=+7{\cdot}89$	$+12476$	
18	Feb. 13	$-60{\cdot}16=$	$+1{\cdot}633$	$+32398$	$=+9{\cdot}47$	$+10027$	
19	1872 Apr. 19	$-26{\cdot}53=$	$+1{\cdot}450$	-1293	$=+3{\cdot}32$	-4220	
20	May 4	$-27{\cdot}55=$	$+1{\cdot}426$	$+7937$	$=+4{\cdot}18$	-4272	
21	1873 June 23	$-35{\cdot}61=$	$+1{\cdot}500$	-1686	$=-1{\cdot}49$	-7297	
22	July 17	$-32{\cdot}43=$	$+1{\cdot}446$	$+14526$	$=-0{\cdot}94$	-6938	
23	1874 Nov. 12	$-132{\cdot}25=$	$+2{\cdot}004$	$+17358$	$=-10{\cdot}79$	$+19646$	
24	1876 Mar. 30	$-43{\cdot}45=$	$+1{\cdot}492$	$+2164$	$=+7{\cdot}65$	-2284	
25	Apr. 19	$-41{\cdot}45=$	$+1{\cdot}420$	$+14570$	$=+7{\cdot}67$	-2445	
26	1877 June 4	$-41{\cdot}02=$	$+1{\cdot}457$	$+2059$	$=+1{\cdot}15$	-6338	
27	22	$-38{\cdot}80=$	$+1{\cdot}406$	$+12450$	$=+0{\cdot}88$	-6049	
28	1878 Aug. 23	$-93{\cdot}16=$	$+1{\cdot}722$	-7613	$=-11{\cdot}79$	-4033	
29	Sept. 5	$-94{\cdot}60=$	$+1{\cdot}729$	$+9874$	$=-13{\cdot}78$	-3296	
30	21	$-90{\cdot}04=$	$+1{\cdot}637$	$+29940$	$=-13{\cdot}69$	-2138	
31	1880 Feb. 18	$-72{\cdot}86=$	$+1{\cdot}609$	-6716	$=+11{\cdot}69$	$+3200$	
32	Mar. 19	$-71{\cdot}42=$	$+1{\cdot}496$	$+21244$	$=+12{\cdot}49$	$+1676$	
33	1881 May 12	$-45{\cdot}20=$	$+1{\cdot}443$	$+2110$	$=+4{\cdot}46$	-5276	
34	June 1	$-45{\cdot}64=$	$+1{\cdot}382$	$+12562$	$=+4{\cdot}77$	-5011	
35	1882 July 18	$-71{\cdot}79=$	$+1{\cdot}567$	-368	$=-5{\cdot}53$	-7546	
36	Aug. 13	$-69{\cdot}50=$	$+1{\cdot}481$	$+20906$	$=-7{\cdot}46$	-6794	
37	Sept. 18	$-55{\cdot}07=$	$+1{\cdot}188$	$+33701$	$=-10{\cdot}19$	-4053	
38	1884 Jan. 11	$-162{\cdot}40=$	$+1{\cdot}868$	$+12076$	$=+11{\cdot}98$	$+17185$	
39	Feb. 17	$-123{\cdot}40=$	$+1{\cdot}448$	$+49724$	$=+14{\cdot}08$	$+10654$	
40	Mar. 14	$-97{\cdot}10=$	$+1{\cdot}169$	$+48987$	$=+14{\cdot}18$	$+5971$	
41	1885 Mar. 30	$-57{\cdot}77=$	$+1{\cdot}426$	-9646	$=+10{\cdot}97$	-3541	
42	Apr. 20	$-56{\cdot}92=$	$+1{\cdot}457$	$+2821$	$=+8{\cdot}18$	-3959	
43	May 2	$-56{\cdot}92=$	$+1{\cdot}424$	$+9740$	$=+9{\cdot}03$	-3944	
44	19	$-51{\cdot}90=$	$+1{\cdot}332$	$+17066$	$=+7{\cdot}82$	-3628	
45	1886 June 11	$-63{\cdot}79=$	$+1{\cdot}476$	-5791	$=-0{\cdot}90$	-6972	
46	July 1	$-63{\cdot}44=$	$+1{\cdot}474$	$+7829$	$=-1{\cdot}39$	-7016	
47	14	$-63{\cdot}69=$	$+1{\cdot}418$	$+15516$	$=-2{\cdot}38$	-6617	
48	1887 Oct. 14	$-214{\cdot}36=$	$+1{\cdot}958$	-3690	$=-23{\cdot}42$	$+11468$	
49	22	$-213{\cdot}82=$	$+1{\cdot}954$	$+13584$	$=-23{\cdot}48$	$+12421$	
50	31	$-210{\cdot}09=$	$+1{\cdot}901$	$+31203$	$=-23{\cdot}44$	$+13446$	
51	Nov. 16	$-191{\cdot}41=$	$+1{\cdot}714$	$+56008$	$=-21{\cdot}07$	$+14225$	
52	28	$-173{\cdot}55=$	$+1{\cdot}542$	$+66540$	$=-19{\cdot}74$	$+14028$	
53	Dec. 7	$-159{\cdot}18=$	$+1{\cdot}420$	$+69854$	$=-18{\cdot}43$	$+13511$	
54	19	$-147{\cdot}48=$	$+1{\cdot}266$	$+71960$	$=-16{\cdot}40$	$+12388$	

The next step is to eliminate $\delta\rho$ between the two equations corresponding to each normal date; so that we have a single equation of the form

$$\alpha\,\delta l + \beta\,\delta b + \gamma = 0,$$

corresponding to each normal date.

To express the variations of heliocentric longitude and heliocentric ecliptic north polar distance in terms of the variations of the elements of orbit, the following formulæ have been used :—

$$\begin{aligned}
\delta l = {} & \frac{\cos i\, a^2}{\cos^2 B\, R^2} \cos\phi \{(t-t_0)\,\delta n + \delta\epsilon\} \\
& + \frac{\cos i}{\cos^2 B}\left(1 - \frac{a^2}{R^2}\cos\phi\right)\delta\pi \\
& + \left(1 - \frac{\cos i}{\cos^2 B}\right)\delta\Omega \\
& - \tan B \cos(l-\Omega)\,\delta i \\
& + \frac{\cos i}{\cos^2 B}\left(\frac{2}{\cos\phi} + \tan\phi\cos v\right)\sin v\,\delta\phi,
\end{aligned}$$

$$\begin{aligned}
-\delta b = {} & \sin i \cos(l-\Omega)\frac{a^2}{R^2}\cos\phi\{(t-t_0)\delta n + \delta\epsilon\} \\
& + \sin i \cos\left(l-\Omega\right)\left(1 - \frac{a^2}{R^2}\cos\phi\right)\delta\pi \\
& - \sin i \cos(l-\Omega)\,\delta\Omega \\
& + \sin(l-\Omega)\,\delta i \\
& + \sin i \cos(l-\Omega)\left\{\frac{2}{\cos\phi} + \tan\phi\cos v\right\}\sin v\,\delta\phi.
\end{aligned}$$

where

l is the heliocentric longitude,
B ,, ,, latitude,
b ,, ,, ecliptic north polar distance,
a ,, semi-axis major of orbit,
R ,, radius vector of planet,
v ,, true anomaly of planet,
$t-t_0$,, time in days from 1876, March $19^d{\cdot}0$

and

n, ϵ, π, Ω, i and e ($=\sin\phi$) are the elements of the orbit.

The required quantities have been taken from, or have been computed from quantities taken from, the *Nautical Almanacs* of the different years.

By the substitution of the values of δl and δb thus obtained in the equations of the form $\alpha\delta l + \beta\delta b + \gamma = 0$, the following equations of condition (where $\delta N = 1000\,\delta n$), connecting the errors of the six elements, have been formed. The adopted weight is simply the number of observations on which each equation depends.

These equations of condition have been solved by the method of least squares, and the subjoined normal equations formed; the values of the six unknown quantities derived from which have been substituted in the equations of condition, and the residuals (given in the last column) computed.

It is to be observed that in the equations of condition the coefficients are logarithms, whilst in the normal equations the coefficients are natural numbers.

Equations of Condition.

No.								Weights.	Residuals.
1	$-0.61839\delta N$	$+9.98498\delta\epsilon$	$+9.73320\delta\pi$	$-8.65321\delta\Omega$	$+9.64542\delta i$	$-9.99651\delta\phi$	$+\ 8''.46=0$	5	$+\ 0''.64$
2	−0.61209	+9.98000	+9.71600	−8.76343	+0.25600	−0.02857	+ 8.00 = 0	5	+ 0.56
3	−0.59415	+9.96332	+9.69108	−8.69897	+0.45697	−9.07188	+ 6.38 = 0	2	− 0.17
4	−1.17541	+0.58984	−9.95999	−0.08243	−9.74974	−0.76864	+ 49.57 = 0	4	+ 1.61
5	−9.73277	+9.14922	−8.55630	+0.22608	+9.60853	−8.86923	− 7.46 = 0	8	− 2.10
6	−2.02732	+1.44458	−0.88694	−1.26475	+0.22246	−1.53596	+381.90 = 0	8	− 0.53
7	−0.80576	+0.28646	+9.10037	−9.68753	−9.84323	+0.61857	+ 34.00 = 0	5	+ 4.69
8	+1.43502	−0.91913	−9.99782	+1.03238	+1.02094	−1.26002	−157.92 = 0	5	+ 7.02
9	−0.39699	+9.92840	+9.73159	+8.67210	−9.84011	+8.97772	+ 15.68 = 0	1	+ 1.01
10	−0.42049	+9.96332	+9.76864	−8.81291	+9.37107	+8.57978	+ 16.52 = 0	4	− 0.30
11	−0.43782	+9.98227	+9.78675	−9.16435	−9.99078	−8.17609	+ 18.06 = 0	4	+ 0.34
12	−0.45215	+9.99826	+9.80277	−9.40140	+0.32160	−9.00000	+ 19.63 = 0	4	+ 1.13
13	−0.57634	+0.18949	+9.29003	−9.22789	−9.81823	−0.49415	+ 35.21 = 0	4	+ 1.06
14	−0.51746	+0.13258	+9.13988	+8.85733	+9.82217	−0.43791	+ 29.63 = 0	4	+ 0.89
15	−0.48684	+0.10346	+9.04139	+9.30963	+0.04218	−0.40824	+ 25.88 = 0	3	− 0.64
16	−0.41682	+0.03503	+8.88081	+9.59106	+0.26007	−0.33905	+ 18.80 = 0	3	− 2.48
17	−0.62991	+0.35603	−9.77232	+9.03743	+9.90200	+0.52673	+ 53.73 = 0	1	− 0.63
18	−0.42588	+0.15594	−9.51720	+9.72428	+0.38274	+0.35353	+ 29.56 = 0	1	− 2.37

No.								Weights.	Residuals.
19	$-0·10781\delta N$	$+9·95231\delta\epsilon$	$+9·69461\delta\pi$	$+8·77085\delta\Omega$	$-9·56110\delta i$	$+9·99870\delta\phi$	$+25''·51=0$	3	$+0''·52$
20	$-0·19398$	$+0·04336$	$+9·79727$	$-9·48430$	$+0·03902$	$+0·04610$	$+35·32=0$	5	$+3·36$
21	$-0·03908$	$+0·03902$	$+9·65610$	$-8·67210$	$-9·17319$	$-0·25575$	$+35·95=0$	5	$+0·89$
22	$-9·97839$	$+9·98900$	$+9·56110$	$+9·02531$	$+0·32139$	$-0·23019$	$+30·46=0$	4	$+0·42$
23	$-0·26441$	$+0·57148$	$-0·20167$	$-9·12710$	$+9·69984$	$-9·87795$	$+141·78=0$	5	$+12·45$
24	$+8·13799$	$+0·09656$	$+9·61909$	$-9·23553$	$+9·25763$	$+0·35736$	$+50·70=0$	6	$+2·62$
25	$+8·77875$	$+0·28735$	$+9·85370$	$-0·09026$	$+0·29292$	$+0·53161$	$+87·16=0$	5	$+8·91$
26	$+9·62226$	$+9·97681$	$+9·74663$	$-8·68124$	$+9·41996$	$-9·88252$	$+41·39=0$	4	$+1·49$
27	$+9·64175$	$+9·97909$	$+9·73799$	$-8·97313$	$+0·30146$	$-9·94399$	$+40·61=0$	4	$+0·75$
28	$+0·37027$	$+0·39533$	$-9·59329$	$-9·56937$	$-9·62941$	$-0·63949$	$+115·42=0$	2	$+0·80$
29	$+0·05664$	$+0·10209$	$-9·36922$	$+9·84386$	$+9·87967$	$-0·33082$	$+53·32=0$	4	$-0·72$
30	$-0·24977$	$-0·28780$	$+9·62014$	$+0·49969$	$+0·32899$	$+0·49541$	$-101·67=0$	7	$-0·54$
31	$+0·46994$	$+0·31429$	$-8·68124$	$-9·60853$	$-9·69020$	$+0·60670$	$+97·39=0$	1	$-3·28$
32	$-0·27996$	$-0·11528$	$-8·70757$	$+0·45500$	$+0·34537$	$-0·41714$	$-86·90=0$	6	$-11·85$
33	$+0·24210$	$+9·96802$	$+9·76268$	$-8·80618$	$+9·28103$	$+9·44404$	$+46·98=0$	4	$-1·50$
34	$+0·29128$	$+0·01242$	$+9·81023$	$-9·46687$	$+0·31345$	$+9·34439$	$+57·60=0$	4	$+3·47$
35	$+0·49051$	$+0·12646$	$+9·39620$	$-8·30103$	$+9·12057$	$-0·42243$	$+72·06=0$	3	$-2·25$
36	$+0·32917$	$+9·95999$	$+9·08636$	$+9·65031$	$+0·39777$	$-0·26031$	$46·53=0$	5	$-1·59$

No.								Weights.	Residuals.
37	−9.67748δN	−9.30320δε	−8.07918δπ	+0.14613δ☊	+0.75404δi	+9.60531δφ	− 29″.66 = 0	5	−10″.82
38	+0.89340	+0.43791	−9.95952	+8.57978	+9.90417	+0.49762	+153.98 = 0	1	+ 6.36
39	+0.51212	+0.05115	−9.49554	+9.80346	+0.58501	+0.19700	+ 57.69 = 0	9	− 4.15
40	−9.77789	−9.31387	+8.68124	+0.12287	+0.76738	−9.50379	− 19.24 = 0	2	+ 0.19
41	+0.29959	+9.78104	+9.46389	+9.72509	−0.21245	+9.93095	+ 27.89 = 0	1	− 7.85
42	+0.53986	+0.01870	+9.72591	−9.07555	+9.41330	+0.13226	+ 62.75 = 0	1	− 2.32
43	+0.60823	+0.08565	+9.80414	−9.63448	+0.13194	+0.17551	+ 79.22 = 0	1	+ 2.39
44	+0.66629	+0.14145	+9.87506	−9.90472	+0.46135	+0.19479	+ 88.68 = 0	2	+ 3.21
45	+0.58906	+0.01662	+9.69548	−8.77085	−9.89597	−0.17085	+ 64.54 = 0	2	− 4.25
46	+0.58053	+0.00604	+9.65801	+7.69897	+0.06296	−0.18977	+ 61.89 = 0	5	− 4.45
47	+0.55443	+9.97359	+9.60746	+8.86923	+0.37310	−0.17435	+ 58.11 = 0	3	− 3.11
48	+1.11212	+0.48615	−0.07591	+8.92942	−9.56348	−0.36642	+206.82 = 0	5	− 3.01
49	+1.17566	+0.54888	−0.14613	−9.26951	+0.60531	−0.38881	+239.50 = 0	5	− 3.90
50	+1.21383	+0.58614	−0.19089	−9.60638	+0.07882	−0.37401	+264.49 = 0	2	− 1.41
51	+1.23252	+0.60325	−0.21880	−9.80754	+0.40037	−0.27784	+274.37 = 0	2	− 4.62
52	+1.22025	+0.58973	−0.21165	−9.85612	+0.52595	−0.15229	+267.18 = 0	3	− 1.84
53	+1.20238	+0.57089	−0.19645	−9.86451	+0.59073	−0.02449	+254.47 = 0	2	− 3.25
54	+1.17507	+0.54245	−0.17114	−9.86747	+0.67265	−9.78319	+242.74 = 0	2	+ 1.46

Normal Equations.

$$
\begin{array}{rrrrrrr}
+102049\cdot921\,\delta N & -23867\cdot784\,\delta\epsilon & +5939\cdot191\,\delta\pi & +17008\cdot243\,\delta\Omega & +900\cdot111\,\delta i & +26541\cdot270\,\delta\phi & -256336\cdot626=0 \\
-23867\cdot784 & +7213\cdot550 & -1803\cdot145 & -4680\cdot678 & +237\cdot062 & -7161\cdot586 & +125136\cdot019=0 \\
+5939\cdot191 & -1803\cdot145 & +576\cdot320 & +1097\cdot233 & -194\cdot644 & +2298\cdot542 & -31261\cdot762=0 \\
+17008\cdot243 & -4680\cdot678 & +1097\cdot233 & +3475\cdot270 & +440\cdot675 & +4103\cdot138 & -72149\cdot789=0 \\
+900\cdot111 & +237\cdot062 & -194\cdot644 & +440\cdot675 & +1492\cdot595 & -1421\cdot714 & +13545\cdot549=0 \\
+26541\cdot270 & -7161\cdot586 & +2298\cdot542 & +4103\cdot138 & -1421\cdot714 & +11956\cdot020 & -102418\cdot880=0
\end{array}
$$

Whence

$\delta N = -\ 7''\cdot160$ with weight 13392·2 $\qquad$ $\delta\Omega = +\ 4''\cdot010$ with weight 156·2

$\delta\epsilon = -38\cdot289$ „ „ 315·3 $\qquad$ $\delta i = +\ 0\cdot351$ „ „ 627·

$\delta\pi = -\ 0\cdot522$ „ „ 66·5 $\qquad$ $\delta\phi = +\ 0\cdot293$ „ „ 1669·1

The corrections to the assumed elements of *Juno* are therefore:—

$dn = +\ 0''\cdot00716$ $\qquad$ $d\Omega = -4''\cdot01$

$d\epsilon = +38\cdot29 + dn \times t$ $\qquad$ $di = -0\cdot35$

$d\pi = +\ 0\cdot52$ $\qquad$ $d\phi = -0\cdot29$

where t is the number of days from 1876 March $19^{d}\cdot0$ G.M.T.

From these corrections to the assumed elements of *Juno*, places near the time of each opposition have been computed for the transit at Greenwich, and from the observed errors of the uncorrected ephemerides (as given in the *Nautical Almanacs* of the different years) the following errors of R.A. and N.P.D. (given in the columns headed "Corrected") have been formed, by comparing the observed and computed places, thus showing the errors of places computed from the corrected elements :—

Date.	$\delta\alpha$. Uncorrected.	$\delta\alpha$. Corrected.	$\delta\alpha \cos\delta$. Corrected.	δ N.P.D. Uncorrected.	δ N.P.D. Corrected.
	s	s	″	″	″
1864 June 8	− 0·53	−0·04	− 0·6	− 1·2	−0·2
1865 Sept. 13	− 1·88	−0·12	− 1·8	+ 2·4	−0·5
1867 Mar. 1	− 1·58	−0·21	− 3·2	− 1·8	+0·1
1868 May 18	− 1·03	−0·02	− 0·3	− 0·9	+0·9
1869 July 30	− 2·06	−0·04	− 0·6	+ 2·0	+0·4
1871 Jan. 27	− 4·01	−0·18	− 2·7	+ 0·9	+1·7
1872 April 21	− 1·75	−0·03	− 0·5	− 4·4	−0·3
1873 June 21	− 2·26	−0·05	− 0·8	− 1·1	−0·4
1874 Nov. 13	− 8·33	−0·70	−10·5	+15·1	−0·3
1876 Mar. 31	− 2·97	−0·18	− 2·7	− 6·6	+0·8
1877 June 6	− 2·60	−0·11	− 1·7	− 3·6	−0·2
1878 Aug. 24	− 6·25	−0·13	− 2·0	+14·5	+0·8
1880 Feb. 21	− 4·98	+0·17	+ 2·6	− 8·9	+0·4
1881 May 10	− 2·91	+0·09	+ 1·4	− 5·5	+1·1
1882 July 16	− 4·63	+0·13	+ 2·0	+ 3·9	−0·3
1884 Jan. 13	−10·15	+0·22	+ 3·3	+ 3·5	+0·9
1885 April 19	− 3·76	+0·25	+ 3·8	− 8·9	+2·0
1886 June 13	− 4·04	+0·27	+ 4·1	− 2·2	−0·2
1887 Oct. 12	−14·02	+0·15	+ 2·3	+36·5	−0·9

The assumed elements of *Juno*, from which the *Nautical Almanac* ephemerides during the years mentioned above have been computed, were deduced by Mr. Hind from observations made near the times of twelve oppositions from 1841–55. The discussion is published in the *Nautical Almanac* for 1859.

An inspection of the errors exhibited in the above table will show that the corrected elements give a very good representation of the observed places during the years 1864–1887, with the exception of those observed in the year 1874, which give a discordant result in R.A.* It will be remarked that the signs of the

* The observed place in this case depends on 5 accordant observations, and leads one to suspect an error—possibly in the application of the perturbations—in the ephemeris. I have made inquiries on this point, but am informed that no such error can be detected. I have not, therefore, felt justified in rejecting this normal place.

errors of R.A. are negative in the earlier years, and positive in the later years, as if the mean motion required still further correction. It must be pointed out, however, that the comparison between the observed and computed places has not been made with any attempt at great accuracy, but rather as a general check on the correctness of the work; and that, in particular, the perturbations have not been recomputed as, of course, they ought to have been had the intention been to deduce a *definitive* orbit of the planet. To that the present paper makes no pretension, the object being merely to deduce corrections to the assumed elements of the orbit so as to obtain elements sufficiently accurate for the purpose of computing ephemerides for some years to come. And, as far as can be seen, that object has been attained.

My thanks are due to the Council of the Royal Society for a grant to defray the expenses of the computations, the results of which are embodied in this paper.

Blackheath:
1890 *June* 4.

On Star-Correction Tables. By W. H. Finlay, M.A.

The computation of star-corrections forms no small part of the reductions for the formation of a catalogue, as every astronomer who has taken part in such work knows well; so that any attempt to shorten the time required for these computations, without sacrificing accuracy or simplicity, is worthy of consideration. I think that this may be satisfactorily done to a considerable extent by tables in the following way. We have for any star—

$$\alpha' - \alpha = f + g \sin(G + \alpha) \tan\delta + h \sin(H + \alpha) \sec\delta,$$

$$\delta' - \delta = \iota \cos\delta + g \cos(G + \alpha) + h \cos(H + \alpha) \sin\delta.$$

Now I put—

$$P = \tfrac{1}{15} \cdot g_0 \sin(G + \alpha) \tan\delta, \qquad P' = g_0 \cos(G + \alpha),$$

$$Q = \tfrac{1}{15} \cdot h_0 \sin(H + \alpha) \sec\delta, \qquad Q' = h_0 \cos(H + \alpha) \sin\delta,$$

and

$$I = \iota \cos\delta,$$

where g_0 and h_0 are arbitrarily chosen constants; and I tabulate the values of P, Q, Q′ for each degree of declination and for every two minutes of time. The value of P′, which is independent of the declination, is given in a separate table for every minute (or smaller interval if desired) of time.